

Crossover from adiabatic to sudden interaction quench in a Luttinger liquid

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Motivated by recent experiments on interacting cold atoms, we analyze interaction quenches in Luttinger liquids (LL), where the interaction is ramped from zero to a finite value within a finite time. The fermionic single particle density matrix reveals several regions of spatial and temporal coordinates relative to the quench time, termed as Fermi liquid, sudden quench LL, adiabatic LL regimes, and a LL regime with time dependent exponent. The various regimes can also be observed in the momentum distribution of the fermions, directly accessible through time of flight experiments. Most of our results apply to arbitrary quench protocols.

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Non-equilibrium dynamics and strong-correlation phenomena in quantum many body systems are topics at the forefront of contemporary physics. When these two fields are combined, namely when strongly correlated systems are driven out-of-equilibrium, we face a real challenge. Experimental advances on ultracold atoms [1] have made the time dependent evolution and detection of quantum many-body systems possible, and in particular, quantum quenching the interactions by means of a Feshbach resonance or time dependent lattice parameters has triggered enormous theoretical [2–5] and experimental [6–9] activity.

Luttinger liquids (LL) are ubiquitous as effective low-energy descriptions of gapless phases in various one-dimensional (1D) interacting systems [10, 11]. In 1D fermions, e.g., Landau's Fermi liquid description breaks down for any finite interaction, and the low-energy physics is described by bosonic collective modes with linear dispersion, and is characterized by anomalous non-integer power-law dependences of correlation functions. The LL similarly arises as the low-energy description of interacting 1D bosons or that of spin chains [10].

A LL could also be driven out of equilibrium through transport [12, 13], e.g., but here we shall concentrate on time dependent changes of the interaction parameter, which is of particular relevance for cold atomic systems. Sudden quenches (SQ) of the interaction in LLs have been considered recently by several authors [14–16], and the idea of the other extreme limit of adiabatic parameter ramps is often invoked, too. However, experimental ramps cannot take infinite time, and are not instantaneous, either. Here we study, how a nonzero quench time, $\tau \neq 0$, influences the final state of the system after a quantum quench. As we show, a finite τ leads to 'heating' effects, and generates excitations in the final state. Moreover, it amounts in the appearance of additional energy ($\sim 1/\tau$) and corresponding length scales: while in certain space-time regions the system reveals universal near-equilibrium (adiabatic) correlations [5], in

other regimes renormalized Fermi liquid (FL) or sudden quench (SQ) behavior is found.

Thus motivated, let us study the LL Hamiltonian[10]

$$H = \sum_{q \neq 0} \omega(q) b_q^\dagger b_q + \frac{g(q, t)}{2} [b_q b_{-q} + b_q^\dagger b_{-q}^\dagger], \quad (1)$$

with $\omega(q) = v|q|$ (v being the bare "sound velocity"), and b_q^\dagger the creation operator of a bosonic density wave. The interaction g is changed from zero to a nonzero value within a quench time τ , $g(q, t) = g_2(q)|q|Q(t)$, with $Q(t)$ encoding the explicit quench protocol, and satisfying $Q(t > \tau) = 1$ and $Q(t < 0) = 0$ [33]. In particular, for a linear quench, $Q(t) = t\Theta(t(\tau - t))/\tau + \Theta(t - \tau)$ with $\Theta(t)$ the Heaviside functions. This setting is very general, and equally applies to switching on the interaction in a spinless fermion system, quenching away from the Tonks-Girardeau limit of a 1D Bose gas [9], or turning on the Z -component of the interaction in an XXZ spin chain[10]. To make contact with previous work [14–16], here we focus on fermionic correlators in detail, but most of our results can be trivially generalized to interacting bosonic systems[18].

We describe time-evolution using the Heisenberg equation of motion, leading to

$$i\partial_t b_q = [b_q, H] = \omega(q) b_q + g(q, t) b_{-q}^\dagger, \quad (2)$$

and similarly, $i\partial_t b_{-q}^\dagger = -\omega(q) b_{-q}^\dagger - g(q, t) b_q$. Solutions of these are of the form

$$b_q(t) = u(q, t) b_q(0) + v^*(q, t) b_{-q}^\dagger(0), \quad (3)$$

where all the time dependence is carried by the prefactors, $u(q, t)$ and $v(q, t)$, and the operators on the r.h.s. refer to non-interacting bosons before the quench. All expectation values are thus taken in terms of the initial density matrix of the latter (or vacuum at $T = 0$). The bosonic nature of the quasiparticles requires

$|u(q, t)|^2 - |v(q, t)|^2 = 1$. From Eqs. (2)-(3), we obtain

$$i\partial_t \begin{bmatrix} u(q, t) \\ v(q, t) \end{bmatrix} = \begin{bmatrix} \omega(q) & g(q, t) \\ -g(q, t) & -\omega(q) \end{bmatrix} \begin{bmatrix} u(q, t) \\ v(q, t) \end{bmatrix}, \quad (4)$$

with the initial condition $u(q, 0) = 1$, $v(q, 0) = 0$. Since both $\omega(q)$ and $g(q, t)$ are even functions of q , $u(q, t)$ and $v(q, t)$ must be so, too. By Eq. (4), all time dependence has been transferred to the Bogoliubov coefficients, and therefore expectation values of the time dependent bosonic modes and non-equilibrium dynamics are calculable using standard techniques developed for equilibrium [10], once the solutions of Eq. (4) are known.

Before discussing a continuous quench, let's see how limiting cases are recovered from Eq. (4). The adiabatic limit follows from replacing $g(q, t)$ with its time independent final value, and looking for the stationary solutions of Eq. (4) at a given energy while ignoring the initial conditions. The SQ limit requires only the replacement of $g(q, t)$ by its final, time independent value, and solving the resulting linear differential equation with the initial conditions satisfied.

With a linear quench, Eq. (4) realizes the non-hermitian Landau-Zener model [19], which can be solved exactly in terms of the parabolic cylinder function. However, the exact solution does not yield an immediate and transparent physical picture. Therefore, to obtain more insight, we assume that $g(q, t) \ll \omega(q)$ (i.e., $g_2(q) \ll v$) for all t and q , and solve Eq. (4) perturbatively in the interaction. To lowest order in $g_2(q)$, we obtain $u(q, t) \approx \exp(-i\omega(q)t)$ and

$$v(q, t > 0) \approx i \int_0^t dt' g(q, t') \exp(i\omega(q)(t - 2t')). \quad (5)$$

Higher order corrections to $u(q, t)$ and $v(q, t)$ are of order $g_2^2(q)$ and $g_2^3(q)$, respectively. We have also checked numerically that Eq. (5) is indeed applicable for any t and τ , as long as $g_2(q) \ll v$ [34]. In the SQ ($\tau \rightarrow 0$) and adiabatic ($\tau \rightarrow \infty$) limits we obtain

$$v(q, t > \tau) \approx \frac{g_2(q)}{2v} \times \begin{cases} 2i \sin(\omega(q)t) & \text{for } \tau \rightarrow 0, \\ -\exp(-i\omega(q)t) & \text{for } \tau \rightarrow \infty, \end{cases} \quad (6)$$

reproducing to lowest order in $g_2(q)$ the SQ results [14, 16] and the equilibrium Bogoliubov transformation [10, 21], respectively.

We are now in position to obtain information about physical observables. We start with the evolution of the total energy of the system. We take the energy of our initial vacuum state to be zero. In the fermionic setting, this corresponds to measuring the energy with respect to the energy of the non-interacting Fermi sea. The expectation value of Eq. (1) is then evaluated in the Heisenberg picture, where the expectation value is taken with the non-interacting ground state, and $v(q, t)$ and $u(q, t)$

as obtained from Eq. (3) keeping track of the time evolution of the system. We thus obtain for $\langle H(t) \rangle$ after the quench ($t > 0$)

$$\langle H \rangle = \sum_{q \neq 0} \omega(q) n_B(q) + (2n_B(q) + 1) \text{Im}[v^*(q, t) \partial_t v(q, t)],$$

with $n_B(q) = 1/(\exp(\omega(q)/T) - 1)$ the Bose function. The expression above is time independent for $t > \tau$, as expected. At $T = 0$ and $t > \tau$, and an interaction of finite range, $g_2(q) = g_2 \exp(-R_0|q|/2)$, we obtain

$$\langle H \rangle = E_{gs} \left[1 - \left(\frac{\tau_0}{\tau} \right)^2 \ln \left(1 + \left(\frac{\tau}{\tau_0} \right)^2 \right) \right] \quad (7)$$

for a linear quench. Here we introduced the microscopic time scale, $\tau_0 \equiv R_0/2v$, and $E_{gs} = -Lg_2^2/4\pi v R_0^2$ is the adiabatic ground state energy shift to lowest order in g_2 , with L the system size. The second term corresponds to quasiparticle excitations resulting from the finite quench speed. In the SQ limit, $\tau \ll \tau_0$, the energy of the system is only slightly shifted[22], $\langle H \rangle = E_{gs}(\tau/\tau_0)^2/2$. This holds true for a general quench, i.e. $\langle H \rangle \sim (\tau/\tau_0)^2$ when $\tau \rightarrow 0$ with a quench dependent coefficient. In the adiabatic limit, $\tau \gg \tau_0$, on the other hand, the excess energy (or “heating”) vanishes as $-2E_{gs} \ln(\tau/\tau_0) \sim \tau_0^2/\tau^2$ in accord with the so-called analytic response of Ref. [23]. This remains valid for general smooth quenches displaying kink(s) (discontinuity in the derivative) and bounded $\partial_t Q(t)$. Smooth quenches without kinks but with bounded $\partial_t Q(t)$ produce also a universal decay as $\sim 1/\tau^2$, while the τ dependence of the heating becomes non-universal for protocols with a diverging $\partial_t Q(t)$ [24]. The crossover between the SQ and adiabatic limits occurs when $\tau \sim \tau_0$, which typically translates to $\tau \sim 1/J$ in an optical lattice, with J the hopping integral in the underlying microscopic lattice Hamiltonian.

In the fermionic context, the structure of the non-equilibrium dynamics can be well demonstrated by means of the fermionic one-particle density matrix. Since the fermion field decomposes to right-going and a left-going parts, $\Psi(x) = e^{ik_F x} \Psi_r(x) + e^{-ik_F x} \Psi_l(x)$, it is enough to concentrate on the right-going part of the density matrix,

$$G_r(x, t) \equiv \langle \Psi_r^\dagger(x, t) \Psi_r(0, t) \rangle,$$

describing excitations around the right Fermi momentum, $k \approx k_F$. The right-going field, $\Psi_r(x)$, can be expressed in terms of the LL bosons as [10]

$$\Psi_r(x) = \frac{\eta_r}{\sqrt{2\pi\alpha}} \exp(i\phi_r(x)), \quad (8)$$

where η_r denotes the Klein factor, and $\phi_r(x) = \sum_{q>0} \sqrt{2\pi/|q|} L e^{iqx - \alpha|q|/2} b_q + h.c.$. Following standard

steps [10, 11], we obtain then

$$G_r(x, t) = G_r^0(x) \exp \left(- \sum_{q>0} \left(\frac{2\pi}{qL} \right) 4 \sin^2 \left(\frac{qx}{2} \right) \times \right. \\ \left. \times [n_B(q) + |v(q, t)|^2 (2n_B(q) + 1)] \right), \quad (9)$$

where $G_r^0(x) = i/(2\pi(x + i\alpha))$ denotes the free fermion propagator, with α an ultraviolet regulator. At $T = 0$, the Bose functions vanish, and only $v(q, t)$, i.e. the mixing between b_q and b_{-q}^+ determines the dynamics.

Let us first discuss the properties of $G_r(x, t)$ long after the quench, $t \gg \tau$. In this limit, we can show [25] that, independently of the quench protocol, $Q(t)$, the one-particle density matrix exhibits universal properties,

$$\frac{G_r(x, t)}{G_r^0(x)} \sim \begin{cases} A(\tau/\tau_0) \left(\frac{R_0}{\min\{|x|, 2vt\}} \right)^{\gamma_{\text{SQ}}} & \text{for } |x| \gg 2v\tau, \\ \left(\frac{R_0}{|x|} \right)^{\gamma_{\text{ad}}} & \text{for } |x| \ll 2v\tau, \end{cases} \quad (10)$$

where $\gamma_{\text{SQ}} = g_2^2/v^2 + \dots$ and $\gamma_{\text{ad}} = g_2^2/2v^2 + \dots$ denote the perturbative sudden quench and adiabatic exponents, respectively. The prefactor $A(\tau/\tau_0)$ depends on the speed of the quench: For a sudden quench it is $A(\tau \ll \tau_0) \sim 1$, while for slower quenches $A(\tau > \tau_0) \sim (\tau/\tau_0)^{\gamma_{\text{ad}}}$.

Thus even for $t \rightarrow \infty$, instead of one single power-law, G_r interpolates between the SQ and adiabatic limits. This is shown in Fig. 1 for a linear quench[25]. Physically, it is easy to understand the cross-over behavior observed in G_r : a finite-time quench is experienced by slow excitations of energy $\omega(q) < 1/\tau$ as a sudden change, while fast excitations with $\omega(q) > 1/\tau$ can adjust to the change in the interaction strength adiabatically. Since high (small) energy excitations determine the short (long) distance correlations, the tail of G_r is governed by the SQ exponent [14], while the short distance behavior is described by the adiabatic exponent. It is remarkable that for slow enough quenches, $\tau \gg \tau_0$, the quench time manifests itself explicitly through an adiabatically *enhanced* prefactor $A \sim (\tau/\tau_0)^{\gamma_{\text{ad}}}$ of the asymptotic tail as also shown in Fig. 1. Thus while the spatial decay of Eq. (10) contains the SQ exponent, its τ dependence reveals the adiabatic LL exponent. For a finite $t \gg \tau$ but $2vt \ll |x|$, $G_r(x, t)$ decays asymptotically as $iZ(t)/2\pi x$, with a finite quasiparticle weight,

$$Z(t \gg \tau, \tau_0) \sim A(\tau/\tau_0) \left(\frac{\tau_0}{t} \right)^{\gamma_{\text{SQ}}}. \quad (11)$$

Thus the exponent observed in $Z(t)$ is identically γ_{SQ} for $t \gg \tau$, but the finite quench time amounts in a quasiparticle weight increased by a factor, $A \sim (\tau/\tau_0)^{\gamma_{\text{ad}}}$ for $\tau \gg \tau_0$. Although these results were obtained perturbatively, they carry over to the non-perturbative limit, too,

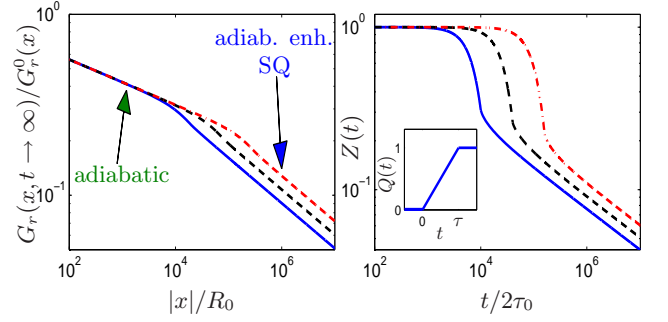


FIG. 1: (Color online) Left: the long time ($\tau \ll t \rightarrow \infty$), steady state limit of the one-particle density matrix is plotted on loglog scale for a linear quench and $2g_2 = v$ as a function of $|x|$, exhibiting the crossover to adiabatic behavior (lower line in Eq. (10)) at small $|x|$ to SQ behavior with adiabatic enhancement (upper line in Eq. (10)) at large $|x|$. The curves are plotted for $\tau/2\tau_0 = 10^4, 4 \times 10^4$ and 16×10^4 from bottom to top in both panels. Right: Landau's quasiparticle weight, $Z(t)$ is plotted on a loglog scale as a function of t , bridging between the weakly interacting Fermi liquid to strongly suppressed $Z \ll 1$ with adiabatic enhancement (Eq. (11)). Inset: the linear quench protocol is shown.

with the only exception that the exponents γ_{SQ} and γ_{ad} must be replaced by their exact value.

All these spatial features appear also in the time-dependent momentum distribution of the fermions, $n(k, t)$, directly measurable through time of flight experiments. In particular, at $T = 0$ and finite $t \gg \tau$, $n(k, t)$ exhibits a jump of size $\sim Z(t)$ at $k = k_F$, while it approximately scales for $|\tilde{k}| \gg 1/2vt$ as

$$n(k) - \frac{1}{2} \sim -\text{sign}(\tilde{k}) \times \begin{cases} A(\tau/\tau_0) |\tilde{k} R_0|^{\gamma_{\text{SQ}}}, & |\tilde{k}| \ll \frac{1}{2v\tau}, \\ |\tilde{k} R_0|^{\gamma_{\text{ad}}}, & |\tilde{k}| \gg \frac{1}{2v\tau}, \end{cases} \quad (12)$$

for $\tilde{k} \equiv k - k_F$, $|\tilde{k}| \ll k_F$, and $t \gg \tau$. Thus the time scale of the quench is also imprinted in the momentum distribution, which also shows a cross-over behavior between the SQ and the adiabatic limits. For adiabatic quenches, $\tau \rightarrow \infty$, we recover the equilibrium LL exponent, while close to k_F , the momentum distribution is enhanced by a factor $A(\tau/\tau_0)$ compared to the SQ behavior[14, 16].

The above analysis can be extended to the short time region, $t \ll \tau$, where the behavior found depends explicitly on the quench protocol[25] as

$$G_r(x, t) \sim G_r^0(x) \left(\frac{R_0}{\min\{|x|, 2vt\}} \right)^{\gamma(t)}, \quad (13)$$

where $\gamma(t) = g_2^2 Q^2(t)/2v^2 + \dots$. For short distances, $|x| \ll 2vt$, the spatial correlations decay with a time-dependent exponent, and this region can thus be characterized as a weakly interacting LL (t-LL). For $|x| \gg 2vt$, on the other hand, similar to $t \gg \tau$, correlations remain almost unaffected by interaction, and a Fermi liquid regime is found. For $t \ll \tau_0$, $Z(t) \simeq 1$ as in the initial

Fermi gas, but for $t \gg \tau_0$ we recover a Fermi liquid behavior with a reduced quasiparticle weight as

$$Z(\tau_0 \ll t \ll \tau) \sim \left(\frac{\tau_0}{t}\right)^{\gamma(t)} \quad (14)$$

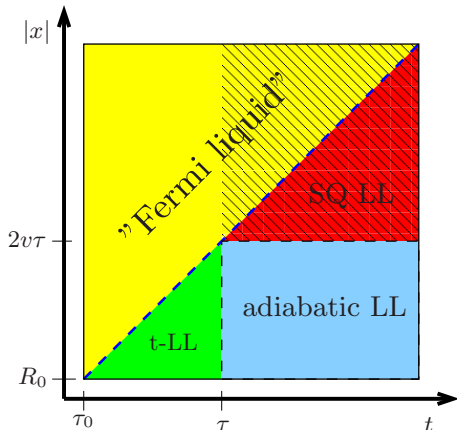


FIG. 2: (Color online) The schematic universal spatial-temporal characteristics of a quenched LL, with the boundaries denoting *crossovers*. In the adiabatic LL regime, the LL exponent of the final state, γ_{ad} governs spatial correlations, while in the SQ LL region, correlations decay with the SQ exponent, $\gamma_{\text{SQ}} > \gamma_{\text{ad}}$. Correlations are adiabatically increased by an amplitude, $A \sim (\tau/\tau_0)^{\gamma_{\text{ad}}}$ in the shaded region. In the Fermi liquid region a time dependent quasiparticle residue is found (see Eq. (14)), while in the time-dependent LL (t-LL) region a quench protocol-dependent weakly interacting LL is found with a time dependent exponent, Eq. (13). The dashed line denotes $|x| = 2vt$, i.e. the light-cone[2]. For $\tau \ll \tau_0$, the SQ physics of Ref. [14, 16] dominates everywhere.

The quasiparticle weight thus slowly decreases during the quench, and excitations remain similar to those in the initial Fermi gas with a reduced weight ($Z < 1$) for $t < \tau$. After the quench, $t > \tau$, the quasiparticle weight continues to decrease as a power-law, and it resembles to an interacting (heavy) Fermi liquid with $1/x$ spatial decay and $Z \ll 1$ quasiparticle residue, Eq. (11), as was also found for sudden quenches [15, 16]. This situation is shown in detail in Fig. 1, where $Z(t)$ is plotted for the special case of a linear quench[25]. Our results are summarized in Fig. 2.

The main effect of finite temperatures ($T > 0$) in Eq. (9) is to introduce a new time or length scale, $1/T$ or v/T , respectively, above which the power law behaviour of LL disappears, and gives way to exponentially suppressed behaviour as $\exp(-cT \max[|x|/v, t])$ with c some constant. Our findings in Fig. 2 survive in the region $(t, \tau, |x|/v) < 1/T$.

Ultracold fermionic gases have been realized using several atoms such as ^{40}K [26–28], ^6Li [29], ^{171}Yb – ^{173}Yb [30], ^{163}Dy [31] and ^{87}Sr [32], and temperatures well in the quantum degeneracy regime were reached ($T < 0.1 E_F$, with E_F the Fermi energy). All these atomic systems

feature tunable interactions, essential to address quench dynamics. Among these, 1D configurations have been realized using ^{40}K [26, 27], ^6Li [29], and the momentum distribution has been measured in time of flight (ToF) experiments in 2D[32] and 3D[28, 30] Fermi gases. Therefore, by applying ToF imaging or momentum resolved rf spectroscopy[26], the observation of the momentum distribution of Eq. (12) is within reach for 1D fermions [26]. Furthermore, the specific momentum distribution of a LL has already been observed in the Tonks-Girardeau limit of 1D Bose systems[9], which exhibit fermionic properties in this strongly interacting regime, and reveal after an interaction quench features similar to the ones found for fermions [18].

In summary, we have studied continuous interaction quenches in LL, bridging smoothly between the SQ and adiabatic limits. The resulting dynamics is largely influenced by the finite quench time for fermions, and in particular, the momentum distribution exhibits a crossover from the adiabatic LL to that of the SQ with the extra adiabatic enhancement factor $(\tau/\tau_0)^{\gamma_{\text{ad}}}$, revealing *both* the equilibrium and SQ LL exponents. A finite quasiparticle residue is retained during the quench, reflecting the Fermi gas nature of the initial state, getting suppressed gradually after the quench. The variety of quench induced phases offers a unique opportunity to design low dimensional correlated states on demand.

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- [1] I. Bloch, *et al.*, Rev. Mod. Phys. **80**, 885 (2008).
- [2] P. Calabrese, and J. Cardy, Phys. Rev. Lett. **96**, 136801 (2006).
- [3] M. Rigol, *et al.*, Nature **452**, 854 (2008).
- [4] J. Dziarmaga, Adv. Phys. **59**, 1063 (2010).
- [5] A. Polkovnikov, *et al.*, arXiv:1007.5331.
- [6] S. Hofferberth, *et al.*, Nature **449**, 324 (2007).
- [7] E. Haller, *et al.*, Nature **466**, 597 (2010).
- [8] T. Kinoshita, *et al.*, Nature **440**, 900 (2006).
- [9] B. Paredes, *et al.*, Nature **429**, 277 (2004).
- [10] T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, Oxford, 2004).
- [11] A. O. Gogolin, *et al.*, *Bosonization and Strongly Correlated Systems* (Cambridge University Press, Cambridge, 1998).
- [12] D. B. Gutman, *et al.*, Phys. Rev. B **81**, 085436 (2010).
- [13] E. Perfetto, *et al.*, Phys. Rev. Lett. **105**, 156802 (2010).
- [14] M. A. Cazalilla, Phys. Rev. Lett. **97**, 156403 (2006).
- [15] G. S. Uhrig, Phys. Rev. A **80**, 061602(R) (2009).
- [16] A. Iucci, and M. A. Cazalilla, Phys. Rev. A **80**, 063619 (2009).

- (2009).
- [17] To avoid instabilities, $v > |g_2(q)|$ is assumed.
- [18] B. Dóra *et al.*, to be published.
- [19] V. S. Shchesnovich, Phys. Lett. A **349**, 398 (2006).
- [20] The solution can easily be extended to include higher powers of $g_2(q)$ for a linear quench.
- [21] J. Sólyom, Adv. Phys. **28**, 201 (1979).
- [22] J.-S. Bernier, *et al.*, arXiv:1010.5251.
- [23] A. Polkovnikov, and V. Gritsev, Nat. Phys. **4**, 477 (2008).
- [24] C. De Grandi, *et al.*, Phys. Rev. B **81**, 224301 (2010).
- [25] See EPAPS Document No. XXX for supplementary material providing further technical details.
- [26] H. Moritz, *et al.*, Phys. Rev. Lett. **94**, 210401 (2005).
- [27] K. Günter, *et al.*, Phys. Rev. Lett. **95**, 230401 (2005).
- [28] M. Köhl, *et al.*, Phys. Rev. Lett. **94**(8), 080403 (2005).
- [29] Y. an Liao, *et al.*, Nature **467**, 567 (2010).
- [30] T. Fukuhara, *et al.*, Phys. Rev. Lett. **98**, 030401 (2007).
- [31] M. Lu, *et al.*, Phys. Rev. Lett. **104**, 063001 (2010).
- [32] B. J. DeSalvo, *et al.*, Phys. Rev. Lett. **105**, 030402 (2010).
- [33] To avoid instabilities, $v > |g_2(q)|$ is assumed.
- [34] The solution can easily be extended to include higher powers of $g_2(q)$ for a linear quench.

SUPPLEMENTARY ONLINE MATERIAL FOR "CROSSOVER FROM ADIABATIC TO SUDDEN INTERACTION QUENCH IN A LUTTINGER LIQUID"

EXACT EVALUATION OF THE BOSONIC CORRELATOR OF EQ. (9) IN THE MAIN TEXT FOR A LINEAR QUENCH

For a linear quench, we can integrate Eq. (5) in the main text to obtain

$$v(q, t) = \frac{g_2(q)|q|}{2\omega^2(q)\tau} [\sin(\omega(q)t) - \omega(q)t \exp(-i\omega(q)t)] \quad (15)$$

for $0 < t < \tau$, and

$$v(q, t) = \frac{ig_2(q)|q|}{4\omega^2(q)\tau} [\exp(i\omega(q)(t - 2\tau)) - \exp(i\omega(q)t) + 2i\omega(q)\tau \exp(-i\omega(q)t)] \quad (16)$$

for $t > \tau$.

The exponent in the one-particle density matrix in Eq. (9) in the main text is then evaluated in closed form for $t > \tau$, $T = 0$ and $L \rightarrow \infty$ as

$$-\sum_{q>0} \left(\frac{2\pi}{qL} \right) 4 \sin^2 \left(\frac{qx}{2} \right) |v(q, t)|^2 = -\frac{g_2^2}{v^2} (I_1(\tau, x, R_0) + I_2(t, \tau, x, R_0)), \quad (17)$$

where

$$I_1(\tau, x, R_0) = \int_0^\infty dq \frac{\exp(-R_0 q)}{q^3 v^2 \tau^2} \sin^2 \left(\frac{qx}{2} \right) [\sin^2(\omega(q)\tau) + (\omega(q)\tau)^2], \quad (18)$$

$$I_2(t, \tau, x, R_0) = \int_0^\infty dq \frac{\exp(-R_0 q)}{q^2 v \tau} \sin^2 \left(\frac{qx}{2} \right) [-\sin(2\omega(q)t) + \sin(2\omega(q)(t - \tau))], \quad (19)$$

which are evaluated as

$$I_1(\tau, x, R_0) = \frac{1}{32(v\tau)^2} \left[\sum_{r,s=\pm 1} \{ \ln(2irv\tau + R_0 + isx)(sx - iR_0 + r2v\tau)^2 \} + \right. \\ \left. + 2 \sum_{s=\pm 1} \{ \ln(R_0 + isx)[(R_0 + isx)^2 + (2v\tau)^2] + \ln(2isv\tau + R_0)(R_0 + 2isv\tau)^2 \} - 4 \ln(R_0)[R_0^2 + (2v\tau)^2] \right] \quad (20)$$

and

$$I_2(t, \tau, x, R_0) = \frac{1}{2} \ln(R_0^2 + 4v^2(\tau - t)^2) - \sum_{s=\pm 1} \frac{1}{4} \ln(R_0^2 + (2v\tau - 2vt - sx)^2) + \\ + \frac{i}{8v\tau} \sum_{s=\pm 1} \left[2(2ivt - sR_0) \ln \left(\frac{sR_0 + 2iv(\tau - t)}{sR_0 - 2ivt} \right) + \sum_{r=\pm 1} (rR_0 - 2ivt + isx) \ln \left(\frac{rR_0 + i(2v\tau - 2vt + sx)}{rR_0 - i(2vt - sx)} \right) \right]. \quad (21)$$

The $0 < t < \tau$ correlator is obtained as

$$-\sum_{q>0} \left(\frac{2\pi}{qL} \right) 4 \sin^2 \left(\frac{qx}{2} \right) |v(q, t)|^2 = - \left(\frac{g_2^2 t}{v\tau} \right)^2 (I_1(t, x, R_0) + I_2(t, x, R_0)). \quad (22)$$

Expanding these in various limits are used to obtain the results cited in the paper, and their general (t, τ, x, R_0) dependence is used to generate Fig. 1. in the main text.

EVALUATION OF THE ASYMPTOTICS OF BOSONIC CORRELATOR OF EQ. (9) IN THE MAIN TEXT FOR GENERAL QUENCH PROTOCOL

In this section, we demonstrate that the asymptotic behavior what we obtained for a linear quench is universal, and arbitrary quench protocols lead to the same behaviour. For arbitrary quench protocol $Q(t)$, the exponent in the one-particle fermionic density matrix of Eq. (9) in the main text at $T = 0$ and $L \rightarrow \infty$ can be rewritten as

$$I = - \sum_{q>0} \left(\frac{2\pi}{qL} \right) 4 \sin^2 \left(\frac{qx}{2} \right) |v(q, t)|^2 = - \frac{g_2^2}{4v^2} \int_0^t dt_1 \int_0^t dt_2 Q(t_1) Q(t_2) \partial_{t_1} \partial_{t_2} f(t_1 - t_2), \quad (23)$$

where

$$f(t) = \ln \left(1 + \frac{x^2}{(R_0 - 2ivt)^2} \right) \quad (24)$$

After partial integrations, it takes the form

$$I = - \frac{g_2^2}{4v^2} \left(Q^2(t) f(0) - 2Q(t) \int_0^t dt_1 Q'(t_1) \text{Re} f(t_1 - t) + \int_0^t dt_1 Q'(t_1) \int_0^t dt_2 Q'(t_2) f(t_1 - t_2) \right). \quad (25)$$

Here, by using $Q(t > \tau) = 1$, the upper limit of integration would reduce to $\min\{t, \tau\}$. However, our considerations remain valid for smooth quench functions as well, reaching 1 only asymptotically, i.e $Q(t \gg \tau) \rightarrow 1$. Let us first consider the properties of the steady state in the limit of $t \gg (\tau, x/v)$, when the middle term in Eq. (25) does not contribute, and $Q(t \gg \tau) \cong 1$. For $t \gg |x/v| \gg \tau$, the first term yields $2 \ln(|x|/R_0)$, while the last integral produces similar spatial decay and the adiabatic enhancement as $2 \ln(|x|/2v\tau)$. The exponent is

$$I = - \frac{g_2^2}{v^2} \ln \left| \frac{x}{\sqrt{R_0 2v\tau}} \right| \text{ for } t \gg |x/v| \gg \tau. \quad (26)$$

In the $t \gg \tau \gg |x/v|$ limit, the last term also vanishes, and we are left with the adiabatic exponent

$$I = - \frac{g_2^2}{2v^2} \ln \left| \frac{x}{R_0} \right| \text{ for } t \gg \tau \gg |x/v|. \quad (27)$$

The other limit of interest is $|x| \gg (v\tau, vt)$, when the exponent simplifies in Eq. (25) with $f(t)$ replaced by $f_1(t) = -2 \ln(R_0 - 2ivt)$. The first term always only contributes with a constant, $-2 \ln(R_0)$. In the limit of $|x| \gg vt \gg v\tau$, $Q(t) = 1$, the third term becomes independent of both x and t and gives rise to the adiabatic enhancement factor as $-2 \ln(\tau)$, and only the second term determines the temporal decay. The exponent is obtained as

$$I = - \frac{g_2^2}{v^2} \ln \left(\frac{t}{\sqrt{\tau_0 \tau}} \right) \text{ for } |x| \gg vt \gg v\tau \quad (28)$$

In the limit of $|x| \gg v\tau \gg vt$, i.e. during the quench, the time dependence of $Q(t)$ is essential. The second term yields $4Q^2(t) \ln(t)$ to leading order, while the third term gives $-1/2$ times the second term. Altogether, the exponent reads as

$$I = - \frac{g_2^2}{2v^2} Q^2(t) \ln \left(\frac{t}{\tau_0} \right) \text{ for } |x| \gg v\tau \gg vt. \quad (29)$$

These agree with the asymptotic expansion of the exact results for a linear quench.
